MATH 135 Winter 2017: Assignment 9   Due at 8:25 a.m. on Wednesday, March 22, 2017

It is important that you read the assignment submission instructions and suggestions available on LEARN.

1. Shade the region of the complex plane defined by $\{2z - 1 \in \mathbb{C} : |\overline{z} - 3| \leq 2\}$. Justify your answer. **March 20 at 1pm: Written in a more standard way, the set above is**

$$\{2z - 1 : z \in \mathbb{C} \text{ and } |\overline{z} - 3| \leq 2\}.$$

2. Let $z_1, z_2 \in \mathbb{C}$ and $r \in \mathbb{R}$. Prove the following identity.

$$|z_1|^2 \cdot [(1-r) \cdot |z_1 - z_2|] + |z_2|^2 \cdot (r \cdot |z_1 - z_2|) = |z_1 - z_2|(|(1-r)z_1 + rz_2|^2 + r \cdot (1-r) \cdot |z_1 - z_2|^2).$$

(When $0 \leq r \leq 1$, this is Stewart's Theorem which you may have seen in the first or second lecture!)

3. Suppose $n \in \mathbb{N}$ and $z \in \mathbb{C}$ with $|z| = 1$ and $z^{2n} \neq -1$. Prove that $\frac{z^n}{1+z^{2n}} \in \mathbb{R}$.

4. Prove there exists $m \in \mathbb{R}$ such that the equation $2z^2 - (3 - 3i)z - (m - 9i) = 0$ has a real root.

5. Let $z = \frac{1}{\sqrt{2}} - i\frac{1}{\sqrt{2}}$. Express z^{26} in standard form. Show your work.

6. Use *De Moivre's Theorem (DMT)* to prove $\cos(5\theta) = 16\cos^5\theta - 20\cos^3\theta + 5\cos\theta$ for all $\theta \in \mathbb{R}$. You may look up and apply the Binomial Theorem to simplify an expression of the form $(x + y)^5$ where x and y are complex numbers.

7. Let z be a nonzero complex number satisfying $z + z^{-1} = 2\cos\left(\frac{\pi}{15}\right)$. Determine the value of $z^{45} + z^{-45}$. Give an exact answer. Show your work.